A High Statistics Lattice Calculation of Quark Masses with a Non-Perturbative Renormalization Procedure.

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We present results of a high statistics study ($O(2000)$ configurations) of the quark masses in the $\overline{MS}$ scheme from Lattice QCD in the quenched approximation at $\beta = 6.0$, $\beta = 6.2$ and $\beta = 6.4$ using both the Wilson and the tree-level improved SW-Clover fermion action. We extract quark masses from the meson spectroscopy and from the axial Ward Identity using non-perturbative values of the renormalization constants. We compare the results obtained with the two methods and we study the $O(a)$ dependence of the quark masses for both actions. Our best results are $m_s^{\overline{MS}}(2\ GeV) = (123 \pm 4 \pm 15)\ MeV$ and $m_c^{\overline{MS}}(2\ GeV) = (1525 \pm 40 \pm 100)\ MeV$.

1. INTRODUCTION

Quark masses are fundamental parameters of the QCD Lagrangian that cannot be measured directly in the experiments, since free quarks are not physical states, and that cannot be predicted by QCD using only theoretical considerations. Up to now QCD sum rules and lattice simulations are the only non-perturbative techniques able to determine the absolute values of the light quark masses. Lattice technique does not require any additional model parameter and is the only procedure that can be systematically improved.

2. QUARK MASSES

The usual on-shell mass definition cannot be used for quark masses and their values depend on the theoretical definition adopted. In the following we will give our final results for the quark masses defined in the $\overline{MS}$ scheme. $m^{\overline{MS}}(\mu)$ is defined by the perturbative expansion of the quark propagator renormalized with the $\overline{MS}$ prescription [1] and it depends on the renormalization scale μ. The symbols used in the following are fully defined in [1,2].

The "bare" lattice quark mass can be determined directly from lattice simulations by fixing the mass of a hadron containing a quark with the same flavour to its experimental value.

Quark masses can also be extracted from the axial Ward Identity. Close to the chiral limit and neglecting terms of $O(a)$, the Ward Identity can be written as

$$Z_A \langle \alpha | \partial^\mu A^a_\mu | \beta \rangle = 2(m - m_c) \frac{Z_P}{Z_S} \langle \alpha | P^a | \beta \rangle \ . \qquad (1)$$

where Z_A, Z_P and Z_S are the renormalization constants of the axial, pseudoscalar and scalar densities and m_c is defined in [1,3]. The standard perturbative approach uses the lattice and the continuum perturbation theory to connect the "bare" lattice quark mass to the $m^{\overline{MS}}(\mu)$ [1,2]. The scale $1/a$, where a is the lattice spacing, of our simulations is $a^{-1} \simeq 2 - 4$ GeV. At these scales we expect small non-perturbative effects. However the "tadpole" diagrams, which are present in the lattice perturbation theory, can give raise to large perturbative corrections and then to large uncertainties in the matching procedure at values of $\beta = 6/g_L^2 = 6.0 - 6.4$.

The non-perturbative renormalization (NP) techniques eliminate these uncertainties [4,5]. Z_A, Z_P and Z_S can be calculated by imposing the renormalization conditions, described in the next section, on the quark states of momentum $p^2 = \mu^2$ and in the Landau gauge[4]. The quark mass in the $\overline{MS}$ scheme is then defined as

$$m^{\overline{MS}}(\mu) = U_m^{\overline{MS}}(\mu, \mu') \left[1 + \frac{\alpha_s(\mu')}{4\pi} C_m^{LAN} \right] \cdot \frac{Z_A}{Z_P^{RI}(\mu' a)} \rho(a) \qquad (2)$$

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Table 1
Summary of the parameters of the runs analyzed in this work.

	Matrix Elements									
	C60a	C60b	C60c	C60d	W60	C62a	W62a	W62b	C64	W64
β	6.0	6.0	6.0	6.0	6.0	6.2	6.2	6.2	6.4	6.4
Action	SW	SW	SW	SW	Wil	SW	Wil	Wil	SW	Wil
# Confs	490	600	200	200	320	250	250	110	400	400
Volume	$18^3 \times 64$	$24^3 \times 40$	$18^3 \times 32$	$16^3 \times 32$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$

	Renormalization Constants					
	C60Z	W60Z	C62Z	W62Z	C64Z	W64Z
β	6.0	6.0	6.2	6.2	6.4	6.4
Action	SW	Wil	SW	Wil	SW	Wil
# Confs	100	100	180	100	60	60
Volume	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$24^3 \times 32$	$24^3 \times 32$

Table 2
Quark Masses from the spectroscopy in MeV. $\overline{MS}$ masses are at $\mu = 2$ GeV.

Run	$m_s(a)$	$m_s^{\overline{MS}}$	$m_c(a)$	$m_c^{\overline{MS}}$
C60a	83(2)	118(7)	-	-
C60b	81(2)	117(7)	-	-
C60c	83(3)	119(7)	-	-
C60d	79(3)	113(7)	-	-
W60	98(2)	133(16)	1335(31)	1612(316)
C62a	83(4)	120(9)	1106(49)	1470(170)
W62a	93(3)	131(15)	1205(24)	1553(249)
W62b	92(4)	129(16)	1206(32)	1560(257)
W64	82(3)	120(16)	-	-
C64	69(3)	103(9)	-	-

where

$$U_m^{\overline{MS}}(\mu,\mu') = (\frac{\alpha_s(\mu)}{\alpha_s(\mu')})^{\gamma^{(0)}/2\beta_0}\,[1+ \\ + \frac{\alpha_s(\mu)-\alpha_s(\mu')}{4\pi}(\frac{\gamma^{(1)}}{2\beta_0} - \frac{\gamma^{(0)}\beta_1}{2\beta_0^2})\Big] \quad (3)$$

and, for large time separations,

$$\rho(a) = \frac{1}{2} M_{PS} \frac{\langle A_0(\tau) P(0)\rangle}{\langle P(\tau)P(0)\rangle} \; . \quad (4)$$

This procedure uses only the continuum perturbation theory to connect the quark mass from the RI-scheme to $\overline{MS}$. The continuum perturbation theory is used at scales $\mu \simeq 2-4$ GeV large enough to avoid non-perturbative effects or higher order corrections.
The non-perturbative approach can be applied directly to the unquenched case [6].

3. NP RENORMALIZATION

For a generic two-quark operator $O_\Gamma(a) = \bar\psi\Gamma\psi$, we define the renormalized operator $O_\Gamma(\mu) = Z^{RI}_{O_\Gamma} O_\Gamma(a)$ by introducing the renormalized constant $Z^{RI}_{O_\Gamma}$ calculated imposing the renormalization condition [4]

$$Z^{RI}_{O_\Gamma}(\mu a) Z_\psi^{-1}(\mu a)\Gamma_{O_\Gamma}(pa)|_{p^2=\mu^2} = 1 \; .$$

$\Gamma_{O_\Gamma}(pa)$ is the forward amputated green function of the bare operator $O_\Gamma(a)$ on off-shell quark states with $p^2 = \mu^2$ in the Landau gauge. For Z_ψ we have used a definition inspired by the Ward Identities

$$Z_\psi = \frac{Tr\left(\sum_{\lambda=1,4}\gamma_\lambda \sin(p_\lambda a) S^{-1}(pa)\right)}{48i\sum_{\lambda=1,4}\sin^2(p_\lambda a)}|_{p^2=\mu^2} \; ,$$

where $S(pa)$ is the quark propagator. These renormalization conditions satisfy the continuum Ward Identity, i.e. $Z_P/Z_S = Z_P^{RI}/Z_S^{RI}$.
This procedure works if μ satisfies the condition $\Lambda_{QCD} \ll \mu \ll 1/a$ to avoid large higher-order perturbative corrections and discretization errors. In the figure 1 we report the matrix element of the renormalized pseudoscalar operator in the chiral limit as a function of the renormalization scale for the runs we have analysed. Data

Table 3
Quark Masses from the Ward Identity in MeV. $\overline{MS}$ masses are at $\mu = 2$ GeV.

Run	$\rho_s(a)$	$m_s^{\overline{MS}}$	$\rho_c(a)$	$m_c^{\overline{MS}}$
C60a	57(2)	129(6)	-	-
C60b	55(1)	125(5)	-	-
C60c	59(2)	132(6)	-	-
C60d	55(2)	127(6)	-	-
W60	77(2)	126(4)	956(20)	1557(52)
C62a	60(4)	127(9)	747(25)	1572(71)
W62a	75(3)	120(6)	936(16)	1502(47)
W62b	71(3)	115(6)	927(22)	1491(55)
W64	69(4)	107(7)	-	-
C64	55(4)	106(8)	-	-

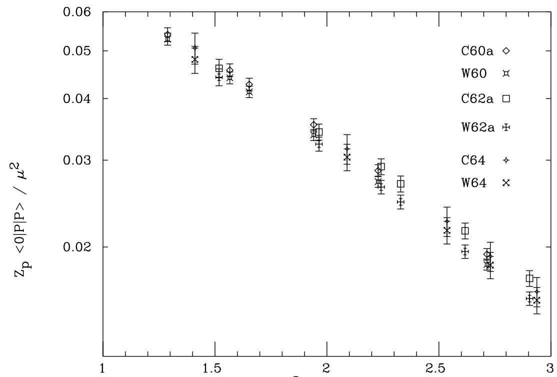

Figure 1. Renormalized pseudoscalar density versus $\log(\mu^2)$ for the runs analysed.

show that the discretization errors are within the statistical errors in the range $\mu a \simeq 1$ where the NP-renormalization is applied to renormalize the quark masses. All details of NP-renormalization will be given in a forthcoming paper [2].

4. RESULTS

We have calculated the quark masses from the meson spectroscopy and from the Ward Identity using different sets of quenched data with $\beta = 6.0$, 6.2 and 6.4 and using either the Wilson or the "improved" SW-Clover action. The parameters used in each simulation are listed in Table 1 and the main results we have obtained are reported in Tables 2 and 3. The data at $\beta = 6.4$ have been used only for an exploratory study. The physical volume and the time extension of the lattice are too small to be considered reliable.
The $\overline{MS}$ mass values in Table 3 show that the non-perturbative approach allows a determination of the quark masses with a smaller error than the standard perturbative approach. In the β range studied, there is *no* statistical evidence for an "a" dependence of the quark masses for the Clover action. For the Wilson action, a mild tendency in the quark masses to decrease with increasing β exists but an extrapolation to the continuum limit is not reliable. The $\overline{MS}$ mass values in Tables 2 and 3 show a very good agreement between the quark masses extracted from the spectroscopy and from the Ward Identity using the non perturbative determinations of the renormalization constants, while the same comparison using the perturbative values of Z_A and Z_P is much poorer [2,7]. We believe that the best estimates for the strange and charm quark masses are obtained from the non-perturbative approach using Wilson and Clover data at $\beta = 6.0$ and $\beta = 6.2$. Our best results are $m_s^{\overline{MS}}(2\ GeV) = (123 \pm 4 \pm 15)\ MeV$ and $m_c^{\overline{MS}}(2\ GeV) = (1525 \pm 40 \pm 100)\ MeV$. The first error is the statistical one and the second is the systematic error estimated extracting the quark masses from different mesons [2]. The values of quark masses we have obtained are in good agreement with previous determinations [1,8–10].

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